The occurrence of an incident is one of the main causes of traffic congestion and delay on urban freeways. Currently, the development of an efficient traffic incident management process is an emerging issue in intelligent transport systems.

The focus of this study is to develop an incident response model for use in an incident management program designed to reduce incident-related traffic congestion and delays. Establishing a fuzzy control system based on a real operation process of incident response would help decrease the workload of operators who are constantly making essential decisions, and would be helpful in developing a more reliable operation procedure. Traffic incident data obtained from traffic detectors and information systems on freeways should be considered in order to determine the best form of operation.

In this study, a fuzzy incident response model is formulated in terms of incident, type of vehicle, type of incident vehicle, location of incident vehicle, and incident service time. In order to analyze the reliability of the proposed model, the application of the model is made using the actual incident data collected on the freeway in the Los Angeles area. The application of the model shows that the fuzzy incident response system is very effective in describing the actual judgment of the incident operators in terms of incident service time.

Traffic congestion is a daily phenomenon in most metropolitan areas, and has negative effects on traffic safety, mobility, and productivity of the transportation system. Congestion is creating bottlenecks on the freeway system and poses serious problems in urban areas. Freeway congestion is made up of two components such as recurring congestion and non-recurring congestion. Recurring congestion is predictable and occurs in locations where the traffic volumes routinely exceed capacity. Non-recurring congestion is very unpredictable and is caused by incidents.

Incidents can be defined as traffic accidents, disabled vehicles, spilled loads, and other random events that reduce the freeway capacity at a specific location. The development of an effective traffic incident management process on freeways has become an important part of transportation system operation. The objective of any incident management strategy is to clear up incidents quickly. Freeway incident management involves a systematic process including incident detection, response, clearance, and recovery to normal traffic conditions. The effectiveness of an incident management program can be evaluated by the amount of reduced time interval between incident occurrence and incident clearance.

The overall duration of an incident depends on the essential incident response service time. The incident response services involve dispatching, responder availability, responder readiness, responder travel distances, incident scene access, communication, traffic management, preplanning and interagency agreements, etc. These services require a significant role of incident operators in order to make decisions in an uncertain incident situation. Uncertainty is the main characteristic of traffic incidents in terms of incident type, location, time, etc.

Uncertainty management is one of the most significant characteristics in the decision making process of fuzzy logic. Human thinking is qualitative, and based on linguistic terms. Fuzzy logic allows ill-measured information to incorporate the experience of a human process operator into the design of the controller. Fuzzy control approaches can easily replicate a controller working under multiple objectives, such as safety, efficiency, and...
stable operation, etc.

In this regard, the purpose of this study is to formulate the Fuzzy Incident Response Model to show that the fuzzy incident response system is effective in describing the actual judgement employed in incident operating. In addition, the application of the model has been made using freeway incident data in order to verify the effectiveness of the model.

Incident management consists of two distinct components: emergency response and traffic response. The emergency response component includes those actions taken by law enforcement and emergency responders who are directly related to resolving the incident and ensuring public safety. The traffic response component includes those actions taken to mitigate the effects of an incident or emergency response both during and after incident occurrence. These activities include road closure, detour routing, and ramp metering, etc.

In general, incident detection and verification require a freeway traffic surveillance system. This system may vary depending on traffic volume, occupancy, and speed, etc. Freeway incidents may be detected by passing travelers (motorists), freeway patrol, or other sources. If an incident is detected by these responders, it is instantly verified in terms of its location and description. For example, if an incident has been reported by motorists at the incident site, the incident operator can quickly verify this report. There is no need to screen out false alarms. Therefore, incident operators are presented with incidents to confirm actions based on a fuzzy system.

### 2.1 Fuzziness of freeway incident management

The development of current freeway traffic management systems is opening many new opportunities for enhanced control strategies involving the traveler, vehicle, and highway. Freeway traffic management systems technology has several components, such as freeway traffic surveillance and control systems, changeable message signs (both fixed and portable), ramp signals, freeway service patrol, highway advisory radio, and call box hotlines.

The management of incidents is one of the major challenges in freeway traffic operations, requiring constant attention and considerable investment. Incident management requires quick and accurate judgment in order to restore the affected area and its surrounding network to normal traffic conditions as quickly as possible. Several methods are currently employed for incident management and automatic incident management techniques are becoming increasingly important for the reduction of traffic delay time caused by incidents.

Freeway incident management deals with various activities of multiple agencies, and the incident management is performed by human operators who consider the various environmental conditions on the freeway. In this regard, uncertainty is the main characteristic of freeway incident management in terms of incident location, time, type, duration, and vehicle type, etc. For example, the detail of incident data collected on the freeway such as the incident type of vehicle (vehicle fire, flat tire, abandoned, mechanical problem, etc.) may hardly be classified.

Currently, many kinds of traffic data are collected during actual freeway incidents. For example, pavement detectors are installed near ramps and along freeway mainlines at 0.5 mile intervals. Generally, the detectors can measure traffic volume, occupancy, and speed. Therefore, data of the queue length is obtained at 0.5 mile intervals. If the actual traffic queue length is 2.2 miles, the incident operator can only obtain traffic information from the queue length interval of 2 miles to 2.5 miles, approximately 2 miles linguistically. Therefore, it is very difficult to obtain accurate traffic flow related data such as traffic volume, occupancy, speed, queue lengths, travel times, etc. In addition, the detailed classification of incident type is very complicated (e.g., disablement, injury accident, non-injury accident, detector malfunction, ramp signal malfunction, etc.).

Table 1 shows an example of the freeway patrol survey incident data categories collected on the Los Angeles freeways in 1995. The data describes the incident type of vehicle, the type of incident vehicle, and the location of incident vehicle. As shown in the Table 1, the detailed information of incident is not reported to the incident management center.

Considering the characteristics of freeway incident and data collecting system, the solution of incident management problems may not be found solely in numerical algorithms, but rather in the application of algorithmic tools guided by human experts using their knowledge and experiences. When an incident is detected, the operator must carry out various procedures to respond to the incident. It is very difficult to improve the incident management process because the actual judgement process of the incident operators cannot be described clearly. In order
to solve these problems, it is necessary to formulate the actual judgement procedure of the incident operators and to develop an automatic decision-making system which is capable of taking the place of the incident operators.

Uncertainty management is one of the most significant characteristics in the decision making process of fuzzy logic. Hence, the application of fuzzy systems emerges as a suitable solution to the incident management problem. Fuzzy systems approach may also be used to study detailed traffic control operator behavior in the context of an incident management environment.

After incident detection and verification have been accomplished, a fuzzy system can be applied to the incident response system. For example, Table 2 below shows three input and one-output variables in fuzzy terms. These variables are derived from an incident operator, and the responses can be selected and recommended by the fuzzy system. The improvement of incident response strategies affects the incident service time interval from occurrence to clearance. The overall incident duration is controlled by the essential incident response services and the fuzzy system describes more effective freeway incident management by obtaining appropriate service time to the scene of an incident.

### 2.2 Previous research

The literature on fuzzy systems has been rapidly expanding in a wider field. In the field of transportation, fuzzy systems have been applied to the areas of planning, control, and operations. However, an application of fuzzy systems has been focused on the areas of traffic signal control and traffic operation. Therefore, an application of a fuzzy system to the incident management area may still be at an early stage.

Fuzzy logic is an extension of conventional logic, and deals effectively with the decision maker’s perception of uncertainty, ambiguity, and vagueness. The vague boundary of the set can be identified by fuzzy sets, which enable the analysis of problems including uncertainty and ambiguity.

The use of a fuzzy controller for traffic signals at a single intersection of two one-way streets was considered by Pappis and Mamdani. They concluded that the fuzzy controller enables further reduction in vehicle delay times than the conventional vehicle-actuated controller. The central idea is that the fuzzy control statements can easily be converted into a decision matrix system. Chen et al. presented an application of fuzzy controller to freeway ramp control at the San Francisco-Oakland Bay Bridge. In their study, the results after the application of the fuzzy controller were compared to that of the existing automatic controller. They examined the six linguistic variables, such as congestion level, change in congestion level, control area, incident, non-incident, and size of control area queue. The testing process of the fuzzy controller presented a possible 40 to 100% savings in passenger-hours.

Another freeway ramp control problem was discussed by Brubaker and Sheerer. Their fuzzy logic system was designed in four stages in order to describe problems and decide fuzzy actions such as: (i) identifying system inputs (speed and density) and the fuzzy range of the inputs (e.g., slow, medium, fast), and establishing a degree of membership functions for each range; (ii) identifying outputs (green light and red light) and the fuzzy range of outputs (e.g., short, medium, long), and establishing a degree of membership functions; (iii) identifying the fuzzy rules that map the inputs to the outputs; and (iv) deciding on a method of combining fuzzy actions into a single, crisp system output. This design minimized the impact of the inflow traffic onto the prevailing freeway traffic. Also, this study shows that the fuzzy control algorithm is very effective in reducing congestion of freeway traffic flow.

Lotan and Koutsopoulos presented the framework for modeling route choice behavior under the provision of real-time traffic information using the concepts of fuzzy set theory, approximate reasoning, and fuzzy control. They used linguistic rules of the form “IF-THEN” to model the decision process. The rules described attitudes towards taking a special route given perceptions (possibly vague) on network attributes. Perceived travel time and traffic information were assumed to be the most
important factors in the route choice process. Then the fuzzy logic route choice model was tested using data from driver simulators. The results presented that human perceptions can be modeled more naturally using linguistic terms and that human behavior can also be modeled more realistically using flexible linguistic rules.

A variety of applications have been tried to evaluate the benefits of fuzzy logic in control systems. One important feature that fuzzy systems incorporate is that of explanation prediction. A concept of fuzzy logic used in fuzzy verification indicates the membership degree to which we can consider to be a prediction of the system.

A fuzzy system model is based on the concepts of input, process structure, and output flow. Fuzzy system models fundamentally fall into two important categories, which differ basically in terms of their ability to represent different types of information. One of the main directions in fuzzy systems is the linguistic approach, based on linguistically described models. The linguistic model depends on the existence of a rule-base and the theory of approximate reasoning. In this study, the linguistic model is extended to the multiple-input, single-output (MISO) form as a tool for complex fuzzy systems.

### 3.1 Classification of input/output linguistic variables of a fuzzy system

As shown in Table 2, three inputs can be defined in the fuzzy incident response model based on Table 1: (i) incident type of vehicle (accident, vehicle fire, abandoned, debris removal, flat tire, mechanical problems, electrical problems, over-heated, shortage of gas, and locked out); (ii) the type of vehicle involved in the incident (big rig, truck, bus, van, pickup, auto, and motorcycle); and (iii) the location of the incident vehicle on the highway (in freeway lanes, on left shoulder, on right shoulder, and on a ramp). It is assumed that an incident operator considers the volumes of all of the inputs. For example, ten incident type of vehicle problems may be classified separately into three categories (Small, Medium, and Large) according to each problem the incident vehicle is experiencing. These input variables are represented by the corresponding three categories as membership functions.

The outputs of the fuzzy system model determine the time of incident manager’s response and the time of incident service. The time of incident service is treated in this model as a fuzzy variable. Five categories of incident service time such as very short, short, medium, long, and very long are represented by appropriate fuzzy sets.

In the fuzzy system, the membership function gives the membership degree ($\mu$) and represents a value from 0 to 1. The membership functions for inputs (SM = Small, ME = Medium, and LA = Large) and outputs (VS = Very Short, SH = Short, ME = Medium, LO = Long, and VL = Very Long) are

<table>
<thead>
<tr>
<th>Input / output variables</th>
<th>Fuzzy terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident Type of Vehicle (IT)</td>
<td>Accident, Vehicle fire, Abandoned, Debris removal (Large (LA))</td>
</tr>
<tr>
<td></td>
<td>Flat tire, Mechanical problem, Electrical problem, Over-heated (Medium (ME))</td>
</tr>
<tr>
<td></td>
<td>Out of gas, Locked out (Small (SM))</td>
</tr>
<tr>
<td>Type of Incident Vehicle (IV)</td>
<td>Big rig, Truck, Bus (Large (LA))</td>
</tr>
<tr>
<td></td>
<td>Van, Pickup (Medium (ME))</td>
</tr>
<tr>
<td></td>
<td>Auto, Motorcycle (Small (SM))</td>
</tr>
<tr>
<td>Location of Incident Vehicle (IL)</td>
<td>In freeway lanes (Large (LA))</td>
</tr>
<tr>
<td></td>
<td>On left shoulder, On right shoulder (Medium (ME))</td>
</tr>
<tr>
<td></td>
<td>On a ramp (Small (SM))</td>
</tr>
<tr>
<td>Output Variable</td>
<td>Incident Service Time (ST)</td>
</tr>
<tr>
<td>Incident Service Time</td>
<td>Very long (VL)</td>
</tr>
<tr>
<td></td>
<td>Long (LO)</td>
</tr>
<tr>
<td></td>
<td>Medium (ME)</td>
</tr>
<tr>
<td></td>
<td>Short (SH)</td>
</tr>
<tr>
<td></td>
<td>Very short (VS)</td>
</tr>
</tbody>
</table>
Very Long) are shown in Table 2. These inputs/outputs can be used to predict the freeway incident service time.

### 3.2 Fuzzy rule base

The fuzzy system is a kind of expert knowledge-based system that contains the control algorithm in a simple rule-base. The fuzzy rule base maps the combination of the inputs to the outputs to decide whether and how to respond to the incident. In the fuzzy system, encoded knowledge is expressed by IF-THEN statements.

The number of rules is equal to the number of input combinations derived from the number of membership functions per input. For instance, if there are three inputs, each having three membership functions, then the number of fuzzy rules would equal to twenty-seven \((3 \times 3 \times 3)\), as given in Table 3.

In this study, the term sets of the input variables of the incident type, incident vehicle, and locations of the disabled vehicles, include the linguistic labels “Small (SM)”, “Medium (ME)”, and “Large (LA)”. Similarly, the term sets of the output variables of the incident service time “ST” include the linguistic labels “Very Short (VS)”, “Short (SH)”, “Medium (ME)”, “Long (LO)”, and “Very Long (VL)”. Considering the above fuzzy terms of input/output incident variables, the fuzzy freeway incident response model can be formulated by the multiple-input and single-output (MISO) system.

### 3.3 Fuzzy algorithm for freeway incident response

Zadeh\(^6,7,8\) developed the idea of formulating fuzzy control algorithms by logical rules. Mamdani and Assilian\(^9\) and Mamdani\(^10\) discussed Zadeh’s concept whereby logical rules with vague predicates can be used to derive inference from vaguely formulated data. A fuzzy control algorithm for multivariable systems proposed by Sanchez\(^11\) and Gupta et al.\(^12\) suggested a solution of multivariable fuzzy control systems.

The analysis and synthesis of a multivariable structure is an important problem in fuzzy control systems. In the multiple-input and single-output systems, the encoded knowledge can be expressed by IF-THEN rules. Since our incident management system has three inputs and one output, the fuzzy system formulation is:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Incident type (IT)</th>
<th>Incident vehicle (IV)</th>
<th>Incident location (IL)</th>
<th>Service time (ST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Small (SM)</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Long (LO)</td>
</tr>
<tr>
<td>2</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>3</td>
<td>Small (SM)</td>
<td>Large (LA)</td>
<td>Very short (VS)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>4</td>
<td>Medium (ME)</td>
<td>Small (SM)</td>
<td>Short (SH)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>5</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Short (SH)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>6</td>
<td>Medium (ME)</td>
<td>Large (LA)</td>
<td>Very short (VS)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>7</td>
<td>Large (LA)</td>
<td>Small (SM)</td>
<td>Short (SH)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>8</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
</tr>
<tr>
<td>9</td>
<td>Large (LA)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
</tr>
<tr>
<td>10</td>
<td>Small (SM)</td>
<td>Small (SM)</td>
<td>Long (LO)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>11</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Very long (VL)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>12</td>
<td>Small (SM)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
</tr>
<tr>
<td>13</td>
<td>Medium (ME)</td>
<td>Small (SM)</td>
<td>Short (SH)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>14</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
</tr>
<tr>
<td>15</td>
<td>Medium (ME)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Very short (VS)</td>
</tr>
<tr>
<td>16</td>
<td>Large (LA)</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Long (LO)</td>
</tr>
<tr>
<td>17</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Short (SH)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>18</td>
<td>Large (LA)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Short (SH)</td>
</tr>
<tr>
<td>19</td>
<td>Small (SM)</td>
<td>Small (SM)</td>
<td>Very long (VL)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>20</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Very long (VL)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>21</td>
<td>Small (SM)</td>
<td>Large (LA)</td>
<td>Long (LO)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>22</td>
<td>Medium (ME)</td>
<td>Small (SM)</td>
<td>Medium (ME)</td>
<td>Long (LO)</td>
</tr>
<tr>
<td>23</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
<td>Short (SH)</td>
</tr>
<tr>
<td>24</td>
<td>Medium (ME)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Long (LO)</td>
</tr>
<tr>
<td>25</td>
<td>Large (LA)</td>
<td>Small (SM)</td>
<td>Long (LO)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>26</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Very long (VL)</td>
<td>Medium (ME)</td>
</tr>
<tr>
<td>27</td>
<td>Large (LA)</td>
<td>Large (LA)</td>
<td>Medium (ME)</td>
<td>Medium (ME)</td>
</tr>
</tbody>
</table>
IF \( U_1 \) is \( A_{(1)} \) and \( U_2 \) is \( A_{(1)} \) and \( U_3 \) is \( A_{(1)} \)
THEN \( V_1 \) is \( B_{(1)} \)
ALSO
...
ALSO
IF \( U_1 \) is \( A_{(0)} \) and \( U_2 \) is \( A_{(1)} \) and \( U_3 \) is \( A_{(0)} \) ......... (1)
THEN \( V_1 \) is \( B_{(0)} \)
ALSO
...
ALSO
IF \( U_1 \) is \( A_{(27)} \) and \( U_2 \) is \( A_{(27)} \) and \( U_3 \) is \( A_{(27)} \)
THEN \( V_1 \) is \( B_{(27)} \).

In the fuzzy system formulation (1), the input variables \( U_1, U_2, \) and \( U_3 \) are the incident type of vehicle, the type of incident vehicle, and the location of the incident vehicle, respectively. Incident service time \( V_1 \) is the output variable of the fuzzy control process. \( A_{(i)}, A_{(i)}, A_{(i)} \) given the crisp ranges of input for each of the fuzzy variables such as large, medium, and small. Output fuzzy term \( B_{(i)} \) is expressed as very long, long, medium, short, and very short, where \( i \) denotes the rule number \((i=1, \ldots, 27)\). These terms are linguistic values (levels) represented as fuzzy subsets of the respective universe of discourse \( X_1, X_2, X_3, \) and \( Y_1 \).

In the case of single-input and single-output fuzzy rule (IF \( U \) is \( A \), THEN \( V \) is \( B \)), the fuzzy relation \( R \) is interpreted as a fuzzy intersection of the fuzzy sets \( A \) and \( B \):

\[
R_i = A_i \cap B_i \quad \text{........................................................ (2)}
\]

\( R_i \) is defined on the Cartesian product space \( X \times Y \), and is characterized by a membership function \( \mu_R \):

\[
R_i(x, y) = A_i(x) \land B_i(y)
\]

and

\[
\mu_R \times Y \to [0,1] \quad \text{........................................................ (3)}
\]

where ‘\( \land \)’ is the min-operator (or intersection operator).

Fuzzy relation \( R \) associated with the individual relations is aggregated using fuzzy union:

\[
R = \bigcup R_i \quad \text{........................................................ (4)}
\]

The membership function of the fuzzy relation \( R \) is:

\[
\mu_R(x,y) = \bigvee R_i(x,y) = \bigvee R_i(x,y) = \bigvee (A_i(x) \land B_i(y)) \quad \text{........................................................ (5)}
\]

where ‘\( \lor \)’ is the max-operator. Therefore, for a given fuzzy relation \( R \) from \( U \) to \( V \) and for given fuzzy values of the input \( A \), the fuzzy output \( B \) is defined by the max-min compositional rule of inference with union operators:

\[
B = A \circ R = A \circ (\bigcup_{i=1}^{n} (A \circ R_i))
\]

or

\[
\mu_B(y) = \max_x \{\min(\mu_A(x), \mu_R(x, y))\} \quad \text{........................................................ (6)}
\]

where the symbol ‘\( \circ \)’ represents a general method for max-min composition of fuzzy relations. This idea can be extended to multiple-input, single-output fuzzy systems. The multivariable linguistic description (1) is expressed as a fuzzy relation \( R \) which is interpreted as the conjunction of the respective reference fuzzy sets:

\[
R_i = A_i \cap B_i \quad \text{........................................................ (7)}
\]

where \( i \) denotes the rule number \((i=1, \ldots, n)\), \( j \) denotes the input variables \((j=1,2,3)\), and \( k \) denotes the output variable \((k=1)\). By applying the rule of inference (6) to each of the subsystems of the three-input one-output system, the output \( B^1 \) is obtained as follows:

\[
B^1 = \bigcup_{i=1}^{n} (A_1 \circ A_2 \circ A_3 \circ R_i) \quad \text{........................................................ (8)}
\]

An analogue to the theory of linear systems, \( B^1 \) can be expressed in the following form of fuzzy equation

\[
B^1 = A_1 \circ R_1 \land A_2 \circ R_2 \land A_3 \circ R_3 \quad \text{........................................................ (9)}
\]

Here \( R \) is the three-dimensional fuzzy matrix; \( B^1 \) is the one-dimensional fuzzy output; and these are decomposed into three one-dimensional fuzzy matrices (i.e., \( R_1^1, R_2^2, R_3^3 \)) and one-dimensional fuzzy output \( B \).

Using the vector-matrix notation, the set of fuzzy equations in (10) permits a simplified decomposed expression for the individual output \( B \) of the multiple variable (three-input, one-output) fuzzy control system:

\[
[B] = \bigcup [A_1, A_2, A_3] \ast \begin{bmatrix}
R_1^1 \\
R_2^2 \\
R_3^3
\end{bmatrix} \quad \text{........................................................ (10)}
\]

where the symbol ‘\( \ast \)’ is the operator (\( \circ, \land \)).

Generally, the membership function of \( B \) can be calculated by the max-min operation. Then the maximum of \( B \) determines the final output \( B \). In this case, we can assume that the first and subsequent rules can be decom-
posed into \( j \) separate sub-relations (see equation 7). To obtain the overall \( j \)th sub-rule, we can unite all contributions (see equation 4):

\[
R^i = \bigcup_{j=1}^{3} R^j_i \quad \text{....................................................} (11)
\]

or, by replacing the union operators with fuzzy max operators:

\[
R^i = \max\{R^1_i(x_j, y), R^2_i(x_j, y), \ldots, R^3_i(x_j, y)\} \quad \text{..........} (12)
\]

where \( x \in A \) and \( y \in B \).

In this study, the final output \( B \) related to a set of three inputs \((A_1, A_2, A_3)\) can be obtained using the max-min operation of all three \((j = 1, 2, 3)\) relations among the three inputs and the relations \( R^1, R^2, \) and \( R^3 \).

\[
B_i = A_1 \circ R^1_i, A_2 \circ R^2_i, A_3 \circ R^3_i \quad \text{....................................................} (13)
\]

where ‘\( \circ \)’ denotes the usual max-min composition.

In a more explicit equation form,

\[
B = \max\{\min\{A_1, R^1(x_1, y)\}, \min\{A_2, R^2(x_2, y)\}, \min\{A_3, R^3(x_3, y)\}\} \quad \text{..........................} (14)
\]

where \( B = \max[B_i] \).

Therefore, the corresponding membership function is defined as follows:

\[
\mu_B(y) = \max\{\mu_{A_1}(x_1) \times \mu_{A_2}(x_2) \times \mu_{A_3}(x_3)\}
\]

\[
\mu_B(x_1, x_2, x_3, y) \quad \text{.................................................................} (15)
\]

\[
x_1 \in A_1, x_2 \in A_2, x_3 \in A_3
\]

Let this fuzzy clause be \( R \). The membership function of fuzzy clause \( R \) is given by:

\[
\mu_R = \max\{\mu_B(x_1, x_2, x_3, y), \mu_{B_1}(x_1, x_2, x_3, y), \ldots, \mu_{B_{27}}(x_1, x_2, x_3, y)\} \quad \text{..........................} (16)
\]

Then the maximum of \( B \) \((i = 1, \ldots, 27)\) determines \( B \), which is calculated as a union:

\[
B = \bigcup_{i=1}^{27} B_i = \max(B_1, B_2, \ldots, B_{27}) \quad \text{....................................................} (17)
\]

Finally, defuzzification of the output is an operation that produces a non-fuzzy output action, a single crisp value \( B^* \), that adequately represents the membership function \( \mu_{agg}(B) \) of an aggregated fuzzy output. In this study, we describe the defuzzification method called mean of maximum (MOM) method which is simple to apply. We define \( B^* \) as the midpoint of the incident service time \([T_1, T_2]\), that is,

\[
B^* = \frac{T_1 + T_2}{2} \quad \text{....................................................} (18)
\]

The three-input, one-output fuzzy system shows how to evaluate the contribution of each component to the overall performance of the system. The block diagram allows a readily visual examination as compared to the linguistic system.

4.1 Study area and data

This section presents the test results of the freeway traffic incident service time algorithm using fuzzy logic. In this study, the freeway incident data collected by the Freeway Service Patrol (FSP) tow truck drivers is applied to the model.

The Los Angeles FSP is the largest dedicated truck patrol program in the USA. The FSP is a joint program of the California Department of Transportation (Caltrans), the California Highway Patrol (CHP), and the Los Angeles Metropolitan Transportation Authority (MTA). The FSP has 164 tow trucks provided by 20 towing contractors patrolling 40 beats with a coverage of 393 centerline miles of freeway in Los Angeles county.

The study area covers 16 miles of the Los Angeles county including the Santa Monica Freeway (I-10), and the area between Bundy Drive on the west and Route 60 at 3rd Street on the east. Two north-south freeways, the San Diego Freeway (I-405) and the Harbor Freeway (I-110), cross the study area. This area nearly coincides with the Los Angeles Smart Corridor project area.

Highway incident related data were collected for 62 weekdays (between January 3, 1995 and March 31, 1995, except holidays), generally from 6:00 to 10:00 a.m. and from 2:30 to 7:00 p.m. and 2,457 incident cases were taken into consideration.

Table 4 shows actual data where survey observation was carried out on the Santa Monica (I-10) Freeway in Los Angeles. Three incident characteristics were used to evaluate the incident databases: (1) ten incident types of vehicle; (2) seven types of incident vehicle; and (3) four locations of incident vehicle.

As shown in Table 4, mechanical problem is the most frequent incident among the type of incident. Also, Auto and right shoulder are the most frequent incident vehicle type and location, respectively.
4.2 Application results and analysis

In this application, since each fuzzy rule is represented by a four-dimensional array, the fuzzy algorithm employed for the three incident characteristics in order to determine the incident service time which is represented by the 27 fuzzy rules (see Table 3). The three-dimensional input corresponding to the one-dimensional output on the incident service conditions is shown in Table 3. For the initial rule matrix, we define the rules to be the intuitive rules resulting from the trivial mapping between the three linguistic label (i.e., SM, ME, and LA) inputs and five linguistic label (i.e., VS, SH, ME, LO, and VL) outputs.

The establishment of the fuzzy relationship is important. In this study, the time of incident service is defined according to incident type (IT), incident vehicle type (IV), and disabled vehicle location (IL). The input given by the user consists of several linguistic variables for certain criteria. For example, “if the degree of incident type is small, and the type and location of incident vehicles is small, then the duration time of incident is medium” is a fuzzy linguistic model, which has three input and five output linguistic variables. The membership functions of these linguistic values are calculated and formulated by a fuzzy incident response model. In the final step, the output is the defuzzification operation which produces a non-fuzzy output action.

The crisp range for incidents’ service time fuzzy variables (very short, short, medium, long, very long) are shown in Table 2. If incident service time is interpreted as a linguistic variable, then its term set ST (incident service time) could be ST = (Very Short, Short, Medium, Long, Very Long) where each term in ST is characterized by a fuzzy set in a universe of discourse \( U_\text{time} = [0,50] \). We might interpret “very short” as “a time below about 5 minutes,” “short” as “a time close to 15 minutes,” “medium” as “a time close to 25 minutes,” “long” as “a time close to 35 minutes,” and “very long” as “a time above about 45 minutes.” Similarly, incident types, incident vehicles, and incident locations are considered by a fuzzy set in a universe of discourse \( U_\text{percent} = [0,100] \).

The first fuzzy rule (see Table 3) is

\[
\text{IF } \mu_\text{small}(1) \text{ and } \mu_\text{small}(5.5) \text{ and } \mu_\text{small}(10.3) \text{ THEN } \mu_\text{medium}(25) = 0.9.
\]

Thus

\[
\mu_R(1,5.5,10.3,25) = \min \{ \mu_\text{small}(1), \mu_\text{small}(5.5), \mu_\text{small}(10.3), \mu_\text{medium}(25) \} = \min\{0,0,0.5,0.9\} = 0
\]

and we decide the maximum value of 27 fuzzy rules as a union (see equation 17).

\[
B = \max\{0,0,0,0,0,0,0,0,0,0.4,0,4,0,4,0,4,0,4,0,4,0,4,0,4,0,4,0,4,0,4,0,0,0,0,0,0\} = 0.4.
\]

Similarly a total of nine variables (small incident type rating, medium incident type rating, large incident type rating, small incident vehicle rating, medium incident vehicle rating, large incident vehicle rating, small incident location rating, medium incident location rating, and large incident location rating) are calculated and formulated by membership functions.
A total of 9 maximum values are applied to fuzzy membership functions to calculate incident service time, by determining the membership functions corresponding to the incident service time for each maximum value. They are defined as 9 membership degrees (µ), that is, three inputs (small; µ = 0, medium; µ = 0.5, and large; µ = 1) and five outputs (very short; µ = 0, short; µ = 0.25, medium; µ = 0.5, long; µ = 0.8, and very long; µ = 0.9).

Figure 1 shows plots of observed values (a) and model value (b) of incident service time in terms of the type of incident vehicle (IV) and incident type of vehicle (IT). In the figure, the X axis (IV) and Y axis (IT) are independent variables and represent the type of incident vehicle and the incident type of vehicle, respectively. However, the Z axis is a dependent variable and represents the incident service time (ST). This figure shows the comparison of distribution between the actual data and the model values using three-dimensional space. In order to represent the application results of the fuzzy incident model using three-dimensional space, two independent input variable combinations can be considered in the model.

As shown in Figure 1, the observed value (a) shows the distribution of incident service time using point markers and the model value (b) shows the distribution of service time using surface plot. Considering the distribution of the model values, we could conclude that the results of the model were in extremely good agreement with observed data.

Figure 2 shows plots of observed value (a) and model value (b) of incident service time in terms of the location of incident vehicle (IL) and the incident type of vehicle (IT).
vehicle (IT). Also, Figure 3 shows plots of observed value (a) and model value (b) of incident service time in terms of the location of incident vehicle (IL) and the type of incident vehicle (IV).

As shown in Figure 2 and Figure 3, the observed value (a) shows the distribution of incident service time using point markers and the model value (b) shows the distribution of service time using surface plot. Considering the distribution of model values, we could also conclude that the results using the model were in extremely good agreement with the observed data.

In addition, we apply three independent input variables (incident type of vehicle, type of incident, location of incident) to the fuzzy incident response model. Although we cannot represent the results using three-dimensional space, Table 5 shows that the estimated value of the model is in the range of 0.3 minutes of the observed data (average error of estimated value is 0.3 minutes). In other words, while the average value of observed incident service time is 15.7 minutes, the estimated value is in the range of 15.4 and 16.0 minutes. Considering these results, the model gives an accuracy of 98.1% for predicting the incident service time (ST) and we can confirm the reliability of the proposed fuzzy incident response model.

In this study, the application of the proposed three input variables may be unrealistic to consider the freeway incident situations. Therefore, further research considering the traffic volume as an input variable for incident service time is necessary. However, this application shows the effectiveness of fuzzy incident response in describing incident service time.

In this study, a fuzzy incident response model is proposed to show that the model is effective in describing the actual judgement employed in incident operating. In addition, an application of the model has been made in order to verify the effectiveness of the proposed model. The results show that the fuzzy system is an effective method for freeway incident management, by getting appropriate responses to the scene of an incident. The performance of the fuzzy system model for freeway incident management can be summarized as follows:

(1) The ability of the fuzzy system model to replace the
The judgement and decision process of human operators in freeway incident management was introduced.

(2) The estimated results of the incident service time in the fuzzy system model show that the model fits the actual incident service time data well.

One of the major limitations of this study relates to the underlying problems in the development of the fuzzy incident response model. Underlying problems which must be improved for more practical use of the model are:

(1) Only three inputs and one output variables are considered in the model. The model does not consider most incident related factors. The traffic volume, the time of day (day or night), the day of the week, weather, and the conditions of the freeway may be important factors for freeway incident management.

(2) The structure of the fuzzy incident response model is not constructed for creating optimal traffic conditions. Some parts of the model structure, such as the production rule, the shape of the membership function, and the transformation of the fuzzy distribution into real freeway traffic conditions should be considered.

However, the most typical point of fuzzy system is to be able to change the structure of the model, such as the definition of fuzzy variables, selection of variables, and formation of fuzzy control rules. Also, the importance of the fuzzy incident response model will be realized to have more practical use when further investigation about the actual freeway traffic condition has been done.